Journal of Structural Geology, Vol. 13, No. 1, pp. 101 to 104, 1991
Printed in Great Britain

0191-8141/91 \$03.00+0.00

The classification of dilatant flow types

C. W. PASSCHIER

Instituut voor Aardwetenschappen, Utrecht University, Budapestlaan 4, P.O. Box 80.021, 3508 TA Utrecht, The Netherlands

(*Received* 26 *February* 1990; *accepted in revised form* 4 *July* 1990)

Abstract—The kinematic vorticity number $\mathcal{W}$ has been used to classify the geometry of two-dimensional, non-dilatant flow as a function of flow vorticity. A similar kinematic dilatancy number $\mathcal{A}$ can be defined, which expresses flow geometry as a function of dilatancy rate. General two-dimensional flow geometry can be described in a simple, non-ambiguous way by just $\mathcal{W}$ and $\mathcal{A}$. Mohr circles for flow can be used to illustrate this classification method.

INTRODUCTION

THE fabric geometry in a deformed material depends in part on the geometry of the parent flow, i.e. on the shape of the velocity field during deformation. Definitions of flow geometry are therefore an essential part of studies on deformation in rocks (cf. Ramberg 1975, McKenzie 1979, Freeman 1985, Bobyarchick 1986, Passchier 1986, 1988a). Two-dimensional and 'plane-strain' flow types are most popular in geological modelling. In the absence of dilatation (here used to mean area-change in the plane of the flow), they can be satisfactorily described in terms of pure shear and simple shear components, or more exactly by a vorticity number (Truesdell 1954, Ramberg 1975, Means *et al.* 1980). A classification problem may appear with the introduction of dilatation into models of flow and progressive deformation. Figure 1(a) illustrates progressive non-dilatant deformation by steady-state simple shear flow. Dilatant flow types such as those responsible for progressive deformation in Figs. 1(b) & (c) are less easy to define. Clearly, some parameters must be chosen to describe such flow geometries in an unambiguous way. The four-component velocity gradient tensor **L** gives a complete mathematical description of flow and could serve this purpose. **L**, however, contains information on the orientation of flow eigenvectors and on strain rate besides flow geometry. Two parameters are sufficient to describe flow geometry completely, as explained below.

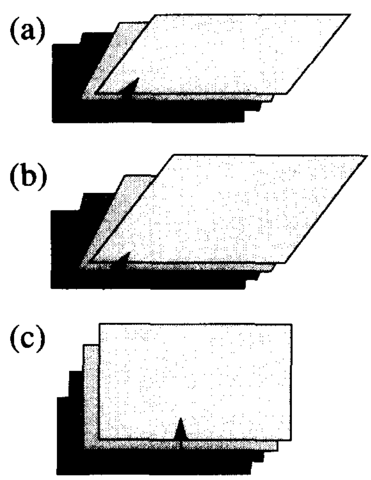

Fig. 1. Three sequences of progressive deformation produced by different steady-state flow types: (a) non-dilatant simple shear; (b) & (c) dilatant flow types.

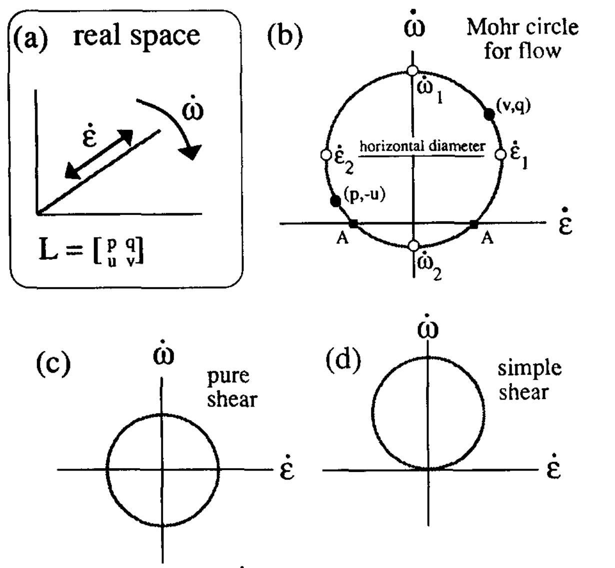

Fig. 2. (a) The angular velocity $\dot{\omega}$ and stretching rate $\dot{\varepsilon}$ of material lines in real space subject to non-dilatant two-dimensional flow as described by the velocity gradient tensor **L** can be plotted to produce (b) a Mohr circle for flow in Cartesian $\dot{\omega} - \dot{\varepsilon}$ space. $\dot{\omega}_1$ and $\dot{\omega}_2$ are directions of maximum and minimum angular velocity; $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$ are directions of maximum and minimum stretching rate; closed circles are points representing co-ordinate axes in real space, constructed by plotting coefficients of **L** in pairs, as indicated. (c) Mohr circle for pure shear flow; (d) Mohr circle for simple shear flow.

DESCRIPTION OF FLOW

Any material line in a plane subject to homogeneous non-dilatant flow has a specific angular velocity ($\dot{\omega}$) and stretching rate ($\dot{\varepsilon}$), depending on its orientation (Fig. 2a). When values for an infinite number of differently oriented material lines are plotted as points in a Cartesian $\dot{\omega} - \dot{\varepsilon}$ space, they define a circle known as the Mohr circle for flow (Fig. 2b) (Lister & Williams 1983, Passchier 1986, 1988b). The angle between material lines measured around this circle (as in any other Mohr circle) is twice the angle measured in real space. Maximum and minimum stretching rates, $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$, occur on orthogonal material lines, defined by a horizontal diagonal through the circle (Fig. 2b); maximum and minimum angular velocities, $\dot{\omega}_1$ and $\dot{\omega}_2$, of material lines occur on orthogonal lines defined by a vertical diameter through the circle (Fig. 2b).

A Mohr circle centred on the origin of the reference frame represents pure shear flow (pure shearing) (Fig. 2c); a circle centred on the vertical axis and transecting the origin represents simple shear flow (simple shearing) (Fig. 2d). A classification of flow in the range between pure and simple shear should therefore be based on the ratio of the vertical position ($\dot{\omega}$) of the circle centre to the size of the circle radius. This ratio can be described as $\mathcal{W}$, a kinematic vorticity number of flow (Truesdell 1954, Means *et al.* 1980, Passchier 1986):

$$\mathcal{W} = \frac{\dot{\omega}_1 + \dot{\omega}_2}{\dot{\omega}_1 - \dot{\omega}_2} = \frac{\dot{\omega}_1 + \dot{\omega}_2}{2r} = \cos\alpha, \tag{1}$$

where $(\dot{\omega}_1 + \dot{\omega}_2)$ is the flow *vorticity* and

$$r = \frac{\dot{\varepsilon}_1 - \dot{\varepsilon}_2}{2} = \frac{\dot{\omega}_1 - \dot{\omega}_2}{2} \tag{2}$$

is the circle radius. α is the angle measured in geometric (real) space between A_1 and A_2, the directions of zero angular velocity (Fig. 3) (Ramberg 1975, Passchier 1988a,b). For pure shear flow $\mathcal{W} = 0$, and for dextral simple shear flow $\mathcal{W} = 1$.

If dilatancy occurs, flow can still be presented as a Mohr circle, but its centre will not lie on the vertical ($\dot{\omega}$) axis (proof in Appendix) (Fig. 2). If the centre lies to the left of the origin, the area of an object deforming by steady-state flow is progressively decreasing; if it lies to the right, it is progressively increasing.

By analogy with $\mathcal{W}$, it is possible to quantify the horizontal deviation of the Mohr circle by a *kinematic dilatancy number* $\mathcal{A}$, the ratio of the horizontal deviation of the circle centre and the circle radius;

$$\mathcal{A} = \frac{\dot{\varepsilon}_1 + \dot{\varepsilon}_2}{\dot{\varepsilon}_1 - \dot{\varepsilon}_2} = \frac{\dot{\varepsilon}_1 + \dot{\varepsilon}_2}{2r} = \cos\beta, \tag{3}$$

where β is the angle measured in geometric (real) space between B_1 and B_2, the directions of zero instantaneous stretching rate (also known as 'lines of no instantaneous longitudinal strain') (Fig. 3). A and B axes are not necessarily orthogonal.

CLASSIFICATION OF DILATANT FLOW

It will be clear from Fig. 3 that the geometry of the Mohr diagram for flow, and therefore of two-dimensional flow itself, can be completely described by two parameters, $\mathcal{W}$ and $\mathcal{A}$. Figure 4 illustrates the full range of possible flow geometries. A axes exist only if

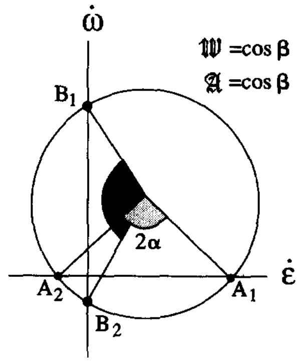

Fig. 3. The geometry of flow, or the orientation of the Mohr circle with respect to its reference frame, can be expressed in terms of $\mathcal{W}$, the kinematic vorticity number, and $\mathcal{A}$, the kinematic dilatancy number. A_1 and A_2 are directions of zero angular velocity; B_1 and B_2 are directions of zero stretching rate.

$-1 < \mathcal{W} < 1$; B axes exist only if $-1 < \mathcal{A} < 1$. Three basic types of flow geometry can be distinguished, depending on the relative position of A and B axes.

Type I: $\mathcal{W}^2 + \mathcal{A}^2 < 1$

These flow types plot inside the circle of Fig. 4 and are characterized by two A and two B axes of opposite sign. Four types of instantaneous deformation of material lines are possible; dextrally rotating and extending; dextrally rotating and shortening; sinistrally rotating and extending; and sinistrally rotating and shortening lines. Most of the flow types responsible for natural deformation with small vorticity numbers and limited dilatancy plot in this category. Pure shear flow is a well-known example (Fig. 4d).

Type II: $\mathcal{W}^2 + \mathcal{A}^2 = 1$

These flow types plot on the circle in Fig. 4. Two or three types of rotating lines exist, and one A and B axis coincide; i.e. there is one line which is neither rotating nor stretching. Flow types in natural shear zones with rigid wall rocks belong to this category. Simple shear flow without dilatation (Fig. 4a) is a well-known example.

Type III: $\mathcal{W}^2 + \mathcal{A}^2 > 1$

These flow types plot outside the circle in Fig. 4. A or B axes have the same sign and three or less types of rotating lines exist (Fig. 4e). Some flow types which either lack A axes (super-simple shear; De Paor 1983), or B axes (pure dilatation) belong to this category. These flow types are probably uncommon in natural deformation.

DISCUSSION

In progressive deformation by a dextral simple shear flow without dilatation (Fig. 1a), one material line is

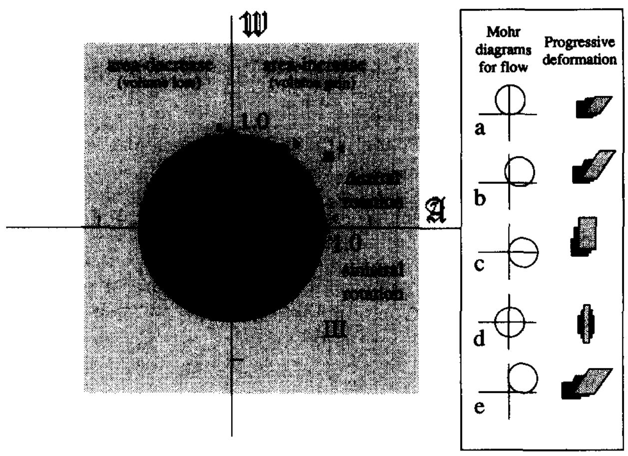

Fig. 4. Flow geometry diagram illustrating all possible two-dimensional flow types as a function of $\mathcal{W}$ and $\mathcal{A}$. I, II and III are main flow categories. Inset at right shows Mohr diagrams and progressive deformation sequences for some specific flow types (a–e).

neither stretching nor rotating, while all other lines are rotating dextrally. If we take the presence of a non-rotating, non-stretching material line as a characteristic of simple shear deformation, the flow type responsible for progressive deformation in Fig. 1(b) could be classified as a simple shear flow with area-increase (McCaig 1987, fig. 8). In this flow type, however, one group of material lines is rotating sinistrally. This is a characteristic of non-dilatant flow types between pure and simple shear; apparently, the addition of a dilatation component to simple shear changes the vorticity number as used here. The flow type responsible for progressive deformation in Fig. 1(c) contains a non-rotating, non-stretching material line as for the flow types of Figs. 1(a) & (b). Nevertheless, it would be classified by most geologists as a pure dilatation, or a pure shear flow with area-increase, rather than a flow type related to simple shear.

Apparently, the flow types responsible for progressive deformation in Fig. 1 cannot satisfactory be classified with the available terminology. In the classification suggested here, they all belong to flow Type II (Figs. 4a–c), but have different vorticity and dilatancy numbers. In order to avoid confusion as outlined above, it may be advantageous to reserve the terms 'simple shear' and 'pure shear' to non-dilatant flow types (Fig. 4; $\mathcal{A} = 0$ axis), and to use $\mathcal{W}$ and $\mathcal{A}$ values to give an exact, non-ambiguous description of flow geometry. Mohr diagrams for flow can easily be constructed from $\mathcal{W}$ and $\mathcal{A}$ values, and can be used to visualize the different flow types. Obviously, many alternative pairs of parameters could be used to classify two-dimensional flow geometry, but the method suggested here has the advantage that it combines the already familiar kinematic vorticity number with a similar parameter, and allows an easy link to Mohr constructions.

Acknowledgements—Chris Talbot proposed use of the term 'kinematic dilatancy number' for the parameter $\mathcal{A}$. A review by Andy Bobyarchick helped to improve the manuscript.

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APPENDIX

Proof for the statement that a Mohr circle for flow centred on the vertical reference axis ($\dot{\omega}$-axis) represents non-dilatant flow

Flow in two dimensions can be represented by the velocity gradient tensor:

$$\mathbf{L} = \begin{pmatrix} p & q \\ u & v \end{pmatrix}. \tag{A1}$$

The Mohr circle for flow (Fig. 2b) can be constructed from tensor coefficients by plotting $(p, -u)$ and (v, q) as Cartesian co-ordinates in the reference frame (Means 1983); the Mohr circle passes through these points and is centred on the diagonal connecting them (Means 1983). This implies (Fig. 3), that the Mohr circle is centred on the vertical axis if $p = -v$.

Integration of **L** gives the position gradient tensor for incremental deformation:

$$\mathbf{F}_i = \int \mathbf{L}\,\mathrm{d}t = \begin{pmatrix} p.\Delta t + 1 & q.\Delta t \\ u.\Delta t & v.\Delta t + 1 \end{pmatrix}, \tag{A2}$$

where Δt is the time over which the incremental deformation operates. If the determinant of $\mathbf{F}_i = 1$, i.e. if:

$$(p.\Delta t + 1)(v.\Delta t + 1) - qu.\Delta t^2 = pv.\Delta t^2 + p.\Delta t + v.\Delta t + 1 - qu.\Delta t^2 = 1 \tag{A3}$$

there is no incremental area change. If $\Delta t \rightarrow 0$, the equation (A3) reduces to

$$p.\Delta t + v.\Delta t = 0, \text{ i.e. } p = -v. \tag{A4}$$

Consequently, a Mohr circle for non-dilatant flow is always centred on the vertical reference axis, and circles which are off-axis represent instantaneous area increase or decrease.